Exercise 50

The table shows values of the viral load $V(t)$ in HIV patient 303, measured in RNA copies/mL, t days after ABT-538 treatment was begun.

t	4	8	11	15	22
$V(t)$	53	18	9.4	5.2	3.6

(a) Find the average rate of change of V with respect to t over each time interval:

(i) $[4, 11]$ (ii) $[8, 11]$

(iii) $[11, 15]$ (iv) $[11, 22]$

What are the units?

(b) Estimate and interpret the value of the derivative $V'(11)$.

Source: Adapted from D. Ho et al., "Rapid Turnover of Plasma Virions and CD4 Lymphocytes in HIV-1 Infection," *Nature* 373 (1995): 123–26.

Solution

Calculate the average rate of change of V with respect to t over each of the time intervals.

(i) $[4, 11]$ $$\frac{V(11) - V(4)}{11 - 4} = \frac{9.4 - 53}{7} = -\frac{218}{35} \approx -6.23 \ \frac{\text{RNA copies/mL}}{\text{day}}$$

(ii) $[8, 11]$ $$\frac{V(11) - V(8)}{11 - 8} = \frac{9.4 - 18}{3} = -\frac{43}{15} \approx -2.87 \ \frac{\text{RNA copies/mL}}{\text{day}}$$

(iii) $[11, 15]$ $$\frac{V(15) - V(11)}{15 - 11} = \frac{5.2 - 9.4}{4} = -\frac{21}{20} = -1.05 \ \frac{\text{RNA copies/mL}}{\text{day}}$$

(iv) $[11, 22]$ $$\frac{V(22) - V(11)}{22 - 11} = \frac{3.6 - 9.4}{11} = -\frac{29}{55} \approx -0.527 \ \frac{\text{RNA copies/mL}}{\text{day}}$$

For the best estimate of the instantaneous rate of change at $t = 11$, take the average of the average rates taken over $[8, 11]$ and $[11, 15]$, the smallest time intervals about $t = 11$.

$$\frac{\left(-\frac{43}{15}\right) + \left(-\frac{21}{20}\right)}{2} = -\frac{47}{24} \approx -1.96 \ \frac{\text{RNA copies/mL}}{\text{day}}$$

This indicates that 11 days after the start of ABT-538 treatment, the HIV viral load in patient 303 is decreasing at a rate of about 1.96 RNA copies/mL per day.